

Өнімділік беттері және
оңтайлы нүктелер

$$\begin{aligned} F(x) = & F(x^*) + \frac{d}{dx}F(x) \Big|_{x=x^*} (x - x^*) \\ & + \frac{1}{2} \frac{d^2}{dx^2}F(x) \Big|_{x=x^*} (x - x^*)^2 + \dots \\ & + \frac{1}{n!} \frac{d^n}{dx^n}F(x) \Big|_{x=x^*} (x - x^*)^n + \dots \end{aligned}$$

$$F(x) = \cos(x).$$

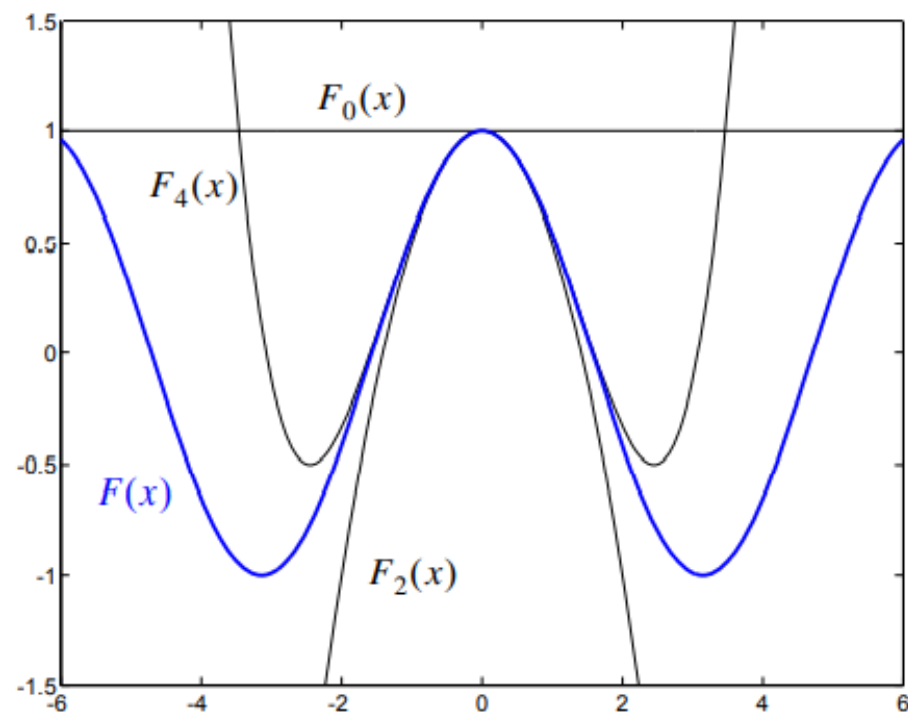
$$F(x) = \cos(x) = \cos(0) - \sin(0)(x-0) - \frac{1}{2}\cos(0)(x-0)^2$$

$$+ \frac{1}{6}\sin(0)(x-0)^3 + \dots$$

$$= 1 - \frac{1}{2}x^2 + \frac{1}{24}x^4 + \dots$$

$$F(x) \approx F_0(x) = 1. \quad F(x) \approx F_2(x) = 1 - \frac{1}{2}x^2.$$

$$F(x) \approx F_4(x) = 1 - \frac{1}{2}x^2 + \frac{1}{24}x^4.$$



Векторлық жағдай

$$F(\mathbf{x}) = F(x_1, x_2, \dots, x_n).$$

$$\begin{aligned} F(\mathbf{x}) = & F(\mathbf{x}^*) + \frac{\partial}{\partial x_1} F(\mathbf{x}) \Big|_{\mathbf{x} = \mathbf{x}^*} (x_1 - x_1^*) + \frac{\partial}{\partial x_2} F(\mathbf{x}) \Big|_{\mathbf{x} = \mathbf{x}^*} (x_2 - x_2^*) \\ & + \dots + \frac{\partial}{\partial x_n} F(\mathbf{x}) \Big|_{\mathbf{x} = \mathbf{x}^*} (x_n - x_n^*) + \frac{1}{2} \frac{\partial^2}{\partial x_1^2} F(\mathbf{x}) \Big|_{\mathbf{x} = \mathbf{x}^*} (x_1 - x_1^*)^2 \\ & + \frac{1}{2} \frac{\partial^2}{\partial x_1 \partial x_2} F(\mathbf{x}) \Big|_{\mathbf{x} = \mathbf{x}^*} (x_1 - x_1^*) (x_2 - x_2^*) + \dots \end{aligned}$$

$$\begin{aligned} F(\mathbf{x}) = & F(\mathbf{x}^*) + \frac{\partial}{\partial x_1} F(\mathbf{x}) \Big|_{\mathbf{x} = \mathbf{x}^*} (x_1 - x_1^*) + \frac{\partial}{\partial x_2} F(\mathbf{x}) \Big|_{\mathbf{x} = \mathbf{x}^*} (x_2 - x_2^*) \\ & + \dots + \frac{\partial}{\partial x_n} F(\mathbf{x}) \Big|_{\mathbf{x} = \mathbf{x}^*} (x_n - x_n^*) + \frac{1}{2} \frac{\partial^2}{\partial x_1^2} F(\mathbf{x}) \Big|_{\mathbf{x} = \mathbf{x}^*} (x_1 - x_1^*)^2 \\ & + \frac{1}{2} \frac{\partial^2}{\partial x_1 \partial x_2} F(\mathbf{x}) \Big|_{\mathbf{x} = \mathbf{x}^*} (x_1 - x_1^*) (x_2 - x_2^*) + \dots \end{aligned}$$

$$\begin{aligned}
 F(\mathbf{x}) &= F(\mathbf{x}^*) + \nabla F(\mathbf{x})^T \Big|_{\mathbf{x}=\mathbf{x}^*} (\mathbf{x} - \mathbf{x}^*) \\
 &\quad + \frac{1}{2} (\mathbf{x} - \mathbf{x}^*)^T \nabla^2 F(\mathbf{x}) \Big|_{\mathbf{x}=\mathbf{x}^*} (\mathbf{x} - \mathbf{x}^*) + \dots
 \end{aligned}$$

$$\nabla F(\mathbf{x}) = \left[\frac{\partial}{\partial x_1} F(\mathbf{x}) \quad \frac{\partial}{\partial x_2} F(\mathbf{x}) \quad \dots \quad \frac{\partial}{\partial x_n} F(\mathbf{x}) \right]^T,$$

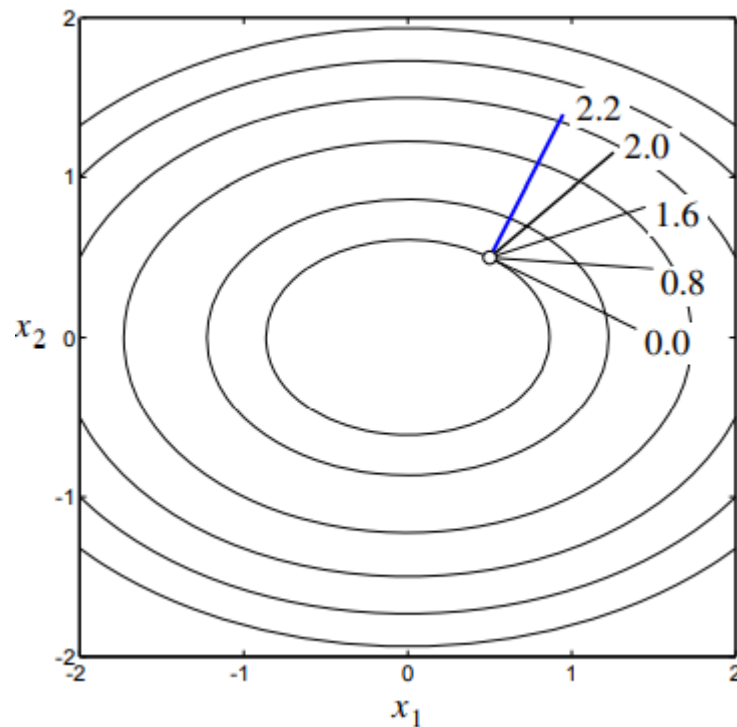
$$\nabla^2 F(\mathbf{x}) = \begin{bmatrix} \frac{\partial^2}{\partial x_1^2} F(\mathbf{x}) & \frac{\partial^2}{\partial x_1 \partial x_2} F(\mathbf{x}) & \dots & \frac{\partial^2}{\partial x_1 \partial x_n} F(\mathbf{x}) \\ \frac{\partial^2}{\partial x_2 \partial x_1} F(\mathbf{x}) & \frac{\partial^2}{\partial x_2^2} F(\mathbf{x}) & \dots & \frac{\partial^2}{\partial x_2 \partial x_n} F(\mathbf{x}) \\ \vdots & \vdots & & \vdots \\ \frac{\partial^2}{\partial x_n \partial x_1} F(\mathbf{x}) & \frac{\partial^2}{\partial x_n \partial x_2} F(\mathbf{x}) & \dots & \frac{\partial^2}{\partial x_n^2} F(\mathbf{x}) \end{bmatrix}$$

$$\frac{\mathbf{p}^T \nabla F(\mathbf{x})}{\|\mathbf{p}\|}, \quad \frac{\mathbf{p}^T \nabla^2 F(\mathbf{x}) \mathbf{p}}{\|\mathbf{p}\|^2}.$$

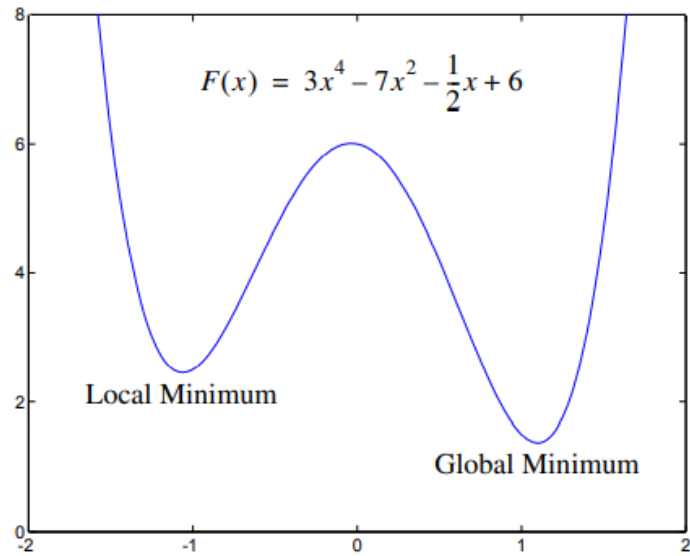
$$F(\mathbf{x}) = x_1^2 + 2x_2^2. \quad \mathbf{x}^* = [0.5 \ 0.5]^T \quad \mathbf{p} = [2 \ -1]^T$$

$$\nabla F(\mathbf{x}) \Big|_{\mathbf{x} = \mathbf{x}^*} = \begin{bmatrix} \frac{\partial}{\partial x_1} F(\mathbf{x}) \\ \frac{\partial}{\partial x_2} F(\mathbf{x}) \end{bmatrix} \Big|_{\mathbf{x} = \mathbf{x}^*} = \begin{bmatrix} 2x_1 \\ 4x_2 \end{bmatrix} \Big|_{\mathbf{x} = \mathbf{x}^*} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}.$$

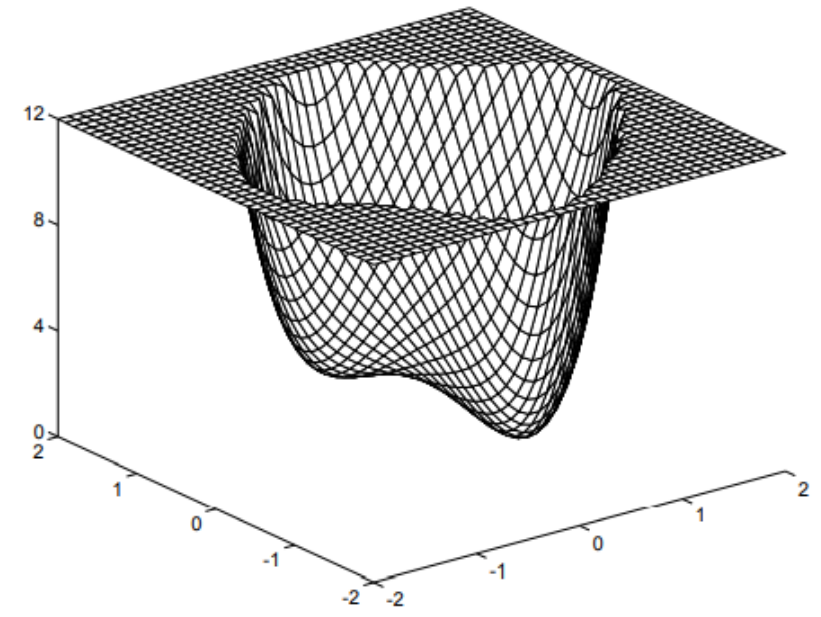
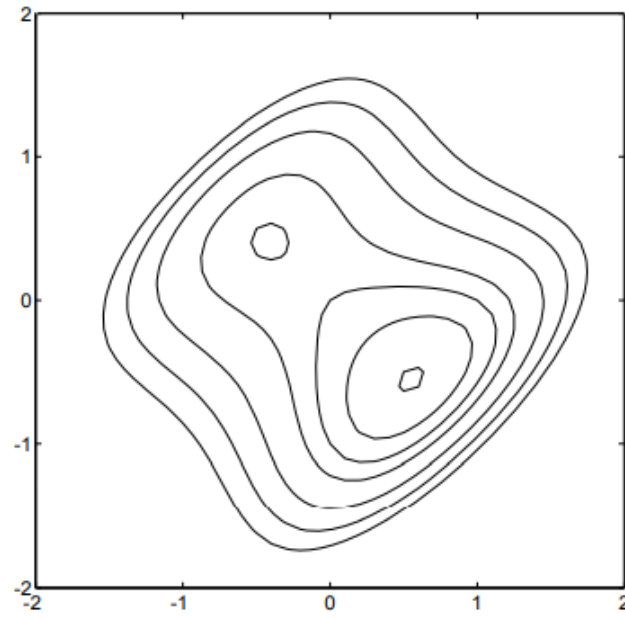
$$\frac{\mathbf{p}^T \nabla F(\mathbf{x})}{\|\mathbf{p}\|} = \frac{[2 \ -1] \begin{bmatrix} 1 \\ 2 \end{bmatrix}}{\left\| \begin{bmatrix} 2 \\ -1 \end{bmatrix} \right\|} = \frac{[0]}{\sqrt{5}} = 0.$$



$$F(x) = 3x^4 - 7x^2 - \frac{1}{2}x + 6.$$



$$F(\mathbf{x}) = (x_2 - x_1)^4 + 8x_1x_2 - x_1 + x_2 + 3.$$



$$F(\mathbf{x}) = F(\mathbf{x}^* + \Delta\mathbf{x}) = F(\mathbf{x}^*) + \nabla F(\mathbf{x})^T \Big|_{\mathbf{x} = \mathbf{x}^*} \Delta\mathbf{x} \\ + \frac{1}{2} \Delta\mathbf{x}^T \nabla^2 F(\mathbf{x}) \Big|_{\mathbf{x} = \mathbf{x}^*} \Delta\mathbf{x} + \dots ,$$

$$\Delta\mathbf{x} = \mathbf{x} - \mathbf{x}^* .$$

Бірінші ретті шарттар

$$F(\mathbf{x}^* + \Delta\mathbf{x}) \cong F(\mathbf{x}^*) + \nabla F(\mathbf{x})^T \Big|_{\mathbf{x} = \mathbf{x}^*} \Delta\mathbf{x}.$$

$$\nabla F(\mathbf{x}) \Big|_{\mathbf{x} = \mathbf{x}^*} = \mathbf{0}.$$

Екінші ретті шарттар

$$F(\mathbf{x}^* + \Delta\mathbf{x}) = F(\mathbf{x}^*) + \frac{1}{2} \Delta\mathbf{x}^T \nabla^2 F(\mathbf{x}) \Big|_{\mathbf{x} = \mathbf{x}^*} \Delta\mathbf{x} + \dots.$$

$$F(\mathbf{x}) = x_1^4 + x_2^2.$$

$$\nabla F(\mathbf{x}) = \begin{bmatrix} 4x_1^3 \\ 2x_2 \end{bmatrix} = \mathbf{0}.$$